### Theorem: Properties of Subsets with Regard to Union and Intersection

Let M and N be sets. Then the following statements are equivalent:NMMN = NN = MM

M ∩ N := {x|x ∈ M ∧ x ∈ N}(N ⊆ MM ∩ Ni) ⇒ (ii)x ∈ M ∩ N x ∈ N x ∈ Mx ∈ Mx ∈ N x ∈ M ∩ Nx ∈ N M ∩ NN ⊆ Proof:

applies. For all therefore also applies. After defining the intersection,

. For all is therefore and therefore

.

For all is by definition and in particular also . Thus

follows ⊆ N.

This means that in total follows.

M ∩ N = N(ii) x ∈ M⇒x ∈ M ∩ N (iii)x ∈ M ∪ N x ∈ M ∪ NM ∩N = N x ∈ Mx ∈ MM ∪ NM ∪ N = Mx ∈ NM ⊆ M ∪ NM ∪ N := {x|x ∈ M ∨ x ∈ N}x ∈ N M ∩ N = N

. According to the definition of the union, . For all is therefore and therefore . also and thus . Thus it follows ⊆ M. For all applies that or . For all is because of

This means that in general it follows that . M ∪ N = Mx ∈ M ∪ N. Assuming that x ∉ M N ⊈ M. Then there is x ∈ N with x ∉ M. It follows that thereM ∪ N = M□

(iii) ⇒ (i)

is with . This leads to a contradiction, because then can-

not apply. Thus, the assumption N ⊈ M is wrong. It therefore follows that N ⊆ M.